Norm inequalities for linear and multilinear singular integrals on weighted and variable exponent Hardy spaces

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Abstract: I will discuss recent work with Kabe Moen and Hanh Nguyen on norm inequalities of the form

$$T : H^{p_1}(w_1) \times H^{p_2}(w_2) \to L^p(w),$$

where T is a bilinear Calderón-Zygmund singular integral operator, $0 < p, p_1, p_2 < \infty$ and

$$\frac{1}{p_1} + \frac{1}{p_2} = \frac{1}{p},$$

the weights w, w_1, w_2 are Muckenhoupt weights, and the spaces $H^{p_i}(w_i)$ are the weighted Hardy spaces introduced by Strömberg and Torchinsky.

We also consider norm inequalities of the form

$$T : H^{p_1(\cdot)} \times H^{p_2(\cdot)} \to L^{p(\cdot)},$$

where $L^{p(\cdot)}$ is a variable Lebesgue space (intuitively, a classical Lebesgue space with the constant exponent p replaced by an exponent function $p(\cdot)$) and the spaces $H^{p_i(\cdot)}$ are the corresponding variable exponent Hardy spaces, introduced by me and Li-An Wang and independently by Nakai and Sawano.

To illustrate our approach we will consider the special case of linear singular integrals. Our proofs, which are simpler than existing proofs, rely heavily on three things: finite atomic decompositions, vector-valued inequalities, and the theory of Rubio de Francia extrapolation.